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Analysis of patients flow in accidents and emergency department.

A case study of 44 Nigerian Reference Hospital, Kaduna.

M.O. Oladejo[1] U.L. Okafor and A.A. Akanbi[2]

Department of Applied Mathematics, Nigerian Defence Academy,

Kaduna State, Nigeria.

ABSTRACT

One of the most important operational issues in health care delivery involves capacity planning such that the goals of efficient resource utilization and providing high quality service are met. The overall aim of this study is to formulate a model that will optimize the exit rate and minimise the death rate over emergency department and intensive care unit in 44 Nigerian Army Reference Hospital, Kaduna. The study also describes the capacity utilization and management of 44 NARHK Emergency Department (ED) and Intensive Care Unit (ICU) using a decomposed queuing model. Based on historical data obtained from the hospitals, the formulae for the measures of performance of a queuing system are adopt from a linked queue network derived by prof. Ikpotokin [17] and modified by Oladejo-Agashua [2], optimal queuing model was determined for each unit. The analysis of the data set obtained in 44 NARHK shows that arrival time was 15.25 per day on the average and the service time was 10.3 patients per day. Queue Model analysis revealed that the intensity was very high at 0.6512. A remedial model was introduced by increasing the number of server to 3 and there was a reduction in the traffic intensity to 0.4635. The proposed model saved extra 0.025826 hours of the patients' time waiting for service in the system. The probability of system been full is much lesser compared to the existing model with increase in the efficiency of the system

Index Terms- Emergency Unit, Intensive Care Unit, Poisson distribution, Exponential distribution, M/M/C queue, Performance measures,

1 Introduction

There are has been series of challenges in the health sector over the past years where quality service delivery has been a bone of issue. Emphasis has been made on the efficient utilization of the available but limited resources due to the inability to afford the expensiveness of the international health standard and its corresponding service level. In Nigeria, during the 2000's 44 Nigerian Army Reference of Hospital experienced restructuring and upgrading to meet international standards. The restructuring and renovation have produced serious overcrowding effect such that patients wait for hours to see doctors in emergency departments (ED).

- M.O Oladejo, Department of Mathematical Sciences, Nigerian Defence Academy, Kaduna. Mooladjo@nda.edu.ng, Mikeoladejo2003@yahoo.com
- A.A. Akanbi, Department of Mathematical Sciences, Nigerian Defence Academy, Kaduna. A.akanbi@nda.edu.ng, harlekzy@gmail.com

But there is an incomplete understanding amongst health policy makers of the limits of the downsizing process and no consensus as to the number of intensive care unit beds necessary to serve a given population (Meticafe, Slogget and Mcpherson0, (1997) [22]. Among the expensive and complex unit of the hospital resources are the ED, and hospital administrators are challenged to meet the demand for intensive care services with an appropriate capacity (Green, 2002) [9].

Researches show that in the ED of the NARHK patients experience longer waiting times to be admitted to or diverted from a unit as it reaches capacity thereby reducing healthcare access to the public, and increasing operational cost to hospitals because of the associated inefficiencies. These major bottlenecks occur at ED, because they have non- interchangeable resources like other few hospital areas. Management of waits, delays and unclogging bottlenecks requires the assessment and improvement of flow between and among various departments in the entire hospital system.

The simple, but elusive goals in health care delivery are "to deliver the right care, to the right patient", "at the right time". "To the right patient", means that the health care delivery system must be able to discriminate among patients with different types and severities of disease so that an individual patient is neither under-or over-treated with an appropriate therapy. "At the right time" means that each patient must have access to care within a time frame that is medically appropriate for his or her illness.

This project surveys the contributions and applications of queuing theory in the field of healthcare processes, in which patients arrive, wait for service, obtain service and then depart.

The mission of ED is to provide timely emergency care to patients in need of medical attention. After attracting the public's attention more than a decade ago, overcrowding in ED has resurfaced as a healthcare crisis in the past few years. Unfortunately, this time overcrowding is even more widespread, and in some places accepted as the standard of care (Henry 2001) [12].

Upon arrival of the patient to the ED, he undergoes registration, diagnostic testing and basic treatment and then is either discharge immediately or admitted for further treatment, the latter if doctors decide on hospitalizing the patient, in which case the patient is then admitted in to the intensive medical unit.

We focus on admitted internal patients, specifically on the process from the decision of hospitalization till admission to the IW. Two main problems could arise in the process: patients' waiting times in the ED for a transfer to the IWs could be long, and patients' allocation to the wards need not be fair.

There is no single answer to the problems that the health care sector faces today. At a good rate, introduction of technology and medical research are been made available, health care delivering process is still inefficient where wait delays and cancelations occur regularly. Hospitals have yielded to these changes by increasing the resources such as more man power per shift, expansion of facilities, and increased number of hospital bed to reduce the delays but this has not absolutely provided solution to the existing problems. But instead, the answer is believed to lie within understanding patient flow as a system and improving ways patients are able to receive timely care (Haraden and Resar, 2004) [11].

Improving the health care process by finding bottlenecks and system failures will involve understanding the system as a whole as patients flow through the system. Understanding the interactions between patients, clinicians, support services, and resources will help show how different departments within the hospital interact (Hall *et al.* 2006).

We believe that one method of improving and understanding the causes of waiting time is through analysis of the queuing parameters and discharge process of patient in ED.

Overcrowding is considered to be a serious public health problem in 91% of surveyed hospital directors and is forecasted to maintain or get worse due to increased closures of EDs, increased ED volumes, growing number of uninsured and decreased reimbursement of uncompensated care (Olshaker and Rathlev 2006).

Overcrowding in the ED creates delays, cause patients to leave without seeing a physician, decrease patient satisfaction, increase patient pain and suffering, and negatively affects the quality of care provided (Fiems *et al*). The inability to transfer emergency patients to inpatient beds is considered to be the most important factor causing overcrowding in the emergency department (Olshaker and Rathlev 2006). Studying the admission and discharge process of patients will also help benefit overcrowding issues within the emergency department.

2. Queuing Theory

The genesis of queuing development was credited to a French man named S. D. Poisson who after some repeated iterations of independents trials successfully described its distribution function to predict the outcome of the independent trials. Agner Krarup Erlang was the telecommunication engineer who is pioneer of queuing theory. He analysed a single facility M/M/s queues called Erlang C mathematical model where arrivals of customers are based on a Poisson process []. Erlang's model inspires mathematicians, scientist, engineers, and Computer Scientist to deal with problems using queuing theory and gave an explicit insight of an additional quality of service to limit waiting times especially in the queue. The removal and insertion from both end is known as Deques or double-ended queues. Since acute patients requires urgent and immediate attention, hospital experience Deques so that the emergency patient can get treated quickly. David G. Kendall also set s major landmark in the history of queuing theory development by introducing the A/B/C notation called the "Kendall's notation" which characterise the queuing parameter and is generally accepted in queuing theory.

2.1 Queuing theory in healthcare and similar environments.

The application of queuing theory in a healthcare setting was rarely used until the pioneering work of Bailey, 1952 appeared. In his research he used queuing theory to develop an out-patient clinic scheduling system that gave acceptable results for patients (in terms of waiting time) and staff (in terms of utilisation). Homogeneity of patients was assumed as far as their service time distributions were

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concerned, and also it was assumed that all patients arrived for appointments on time. In more recent years, a vast number of queuing models have been developed for use in healthcare settings. Soon after, in 1954, a paper entitled "Queuing for Medical Care" was published by Bailey (1952). The author relates his study of an inpatient facility to Erlang's work on telephony by considering patients as telephone calls and hospital beds as telephone channels. The length of stay (LoS) is equivalent to the duration of the call. The author deduced the average waiting time (through Erlang's formula) and calculated the optimal number of beds required in the hospital. In subsequent years and decades, research interest in healthcare modelling through queuing theory has developed and there now exist a multitude of studies.

A considerable body of research has shown that queuing theory can be useful in real-world health-care situations, and some reviews of this work have appeared. Cahill, (1999) reviewed re-search on models for evaluating the impact of bed assignment policies on utilisation, waiting time, and the probability of turning away patients. Nosek and Wilson, (2001) reviewed the use of queuing theory in pharmacy applications with particular attention to improving patient satisfaction. Patient satisfaction is improved by predicting and reducing waiting times and adjusting staffing levels. Miliken *et al*, (1972) presented a brief history of the use of queuing theory in health-care. Green (2006) presented the theory of queuing as applied in healthcare. The relationship between delays, utilisation and the number of servers was discussed, including the basic M/M/c model, its assumptions and extensions, and the applications of the theory to determine the required number of servers. Fomundam and Herrmann, 2007 summarised a range of queuing theory results in the following areas: waiting time and utilisation analysis, system design, and appointment systems. Their goal was to provide sufficient information to analysts who were interested in using queuing theory to model a healthcare process and who wanted to locate the details of relevant models.

3 Queuing model consideration

The data was obtained from 44 NARHK through daily observations at the hospital. Data from the queuing model in the 44 NARHK is analysed. The queuing theory analysis focuses on the system which is in a steady state condition since its behaviour is independent of its initial conditions and of the elapsed time.

Patients have an arrival rate which may be regular or irregular (random arrivals). They form a queue at the service channels. When service has been completed, the patient leaves the system. The seven main elements of a queue system are used to explain the queuing operations in 44 NARHK.

1. **Arrival distribution**: This is the pattern in which patients arrives at the hospital. In hospital patients have a random arrival since they arrive at irregular interval of time. When arrival is random, the Poisson probability distribution will describe the number of patients arriving. The Mean value of arrival rate represented by λ. Thus, λ_n is recorded according to the number of sections (n) the patient has to pass through. Poisson distribution can handle random phenomena wherein it is possible to describe the number of times an event occurs, but not the number of times it does not occur.
2. **Service distribution**: It represents the pattern in which the number of patients leaves the service facility. In 44 NARHK, since the service times are randomly distributed, the exponential probability will best describe the service time. Exponential distribution is encountered in queuing problems as a probability model for service time, which is acquired from the hospital. The average number of patients per unit of time is called the service rate which represents μ.
3. **Service channel:** The 44 NARHK service channel handles a combination of a parallel-series channel. The queuing model of 44 NARHK is considered as a multi-channel (parallel-series) model. It is represented in Figure 3.2 where the existing Model structure of hospital is shown.

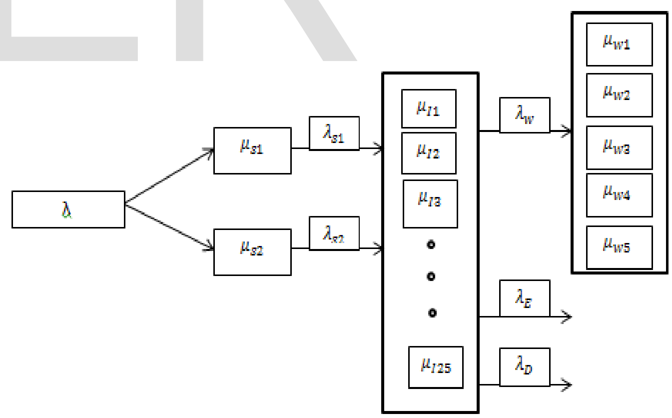

Figure 3.1 Existing healthcare structures at 44 NARHK

Where;

λ is the arrival time at the emergency department.

μ_1 and μ_2 are the service time at the emergency department.

$\lambda 1$ and $\lambda 2$ are the arrival time at the ICU

$\mu_{i\ \forall i=1,2,\cdots,10}$ service rate at the ICU

λ_w is the arrival rate of patients into the IW of the hospital.

λ_E is the exit rate of patients from the ICU

λ_D is the average number of patients that died in the emergency department

As it is seen from figure 3.1, the existing structure is in line with modified Oladejo Agashua Model [2] which is a series of interconnected parallel and series model network.

4 **Service discipline**: service discipline or order of service is the rule by which patients are selected from the service. The 44 NARHK follows a service discipline of priority discipline.

5 **Capacity of the system**: This is based on the arrival source capacity. The 44 NARHK is not affected by the number of patients in the hall because once they are treated, they leave the premises immediately. The maximum number of patients in a system is infinite.

6 **Calling source or population**: The arrival pattern of patients depends upon the source which generates them. 44 NARHK has a large number of patients over time thus, the calling source is infinite.

7 **Patients behaviour**: In 44 NARHK the multi-server model shows that in parallel channels with respect to each series, when patients arrive, he has to wait for the available resources where he is been treated.

The proposed Model Structure for 44 NARHK is represented in Figure 3.3

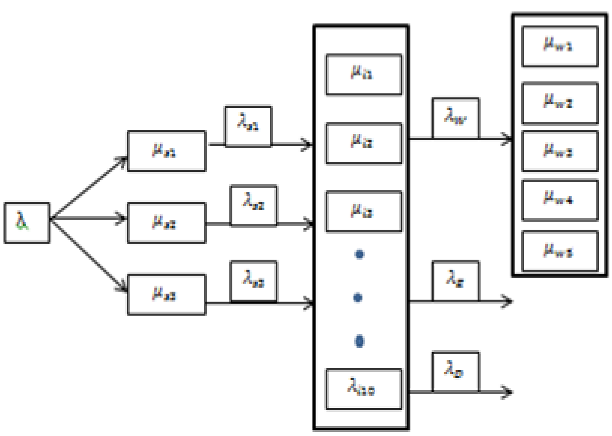

Figure 3.2 Proposed model structures at 44 NARHK

The queuing model that best describe the operation of 44 NARHK is ($M/M/C:PS/S_c/\infty$). For the analysis of the 44 NARHK data, using Oladejo-Agashua modified model, the measure of performance of the system are thus:

1 Expected (average) number of patients in the system:

$$L_s = \sum_{i=1}^{n} \frac{\lambda_i}{\mu_i - \lambda_i} + \frac{\sum_{i=1}^{n} \lambda_i}{\mu_{i-1} - \sum_{i=1}^{n} \lambda_i}$$

2 Expected (average) number of patients in the queue:

$$L_q = \sum_{i=1}^{n} \frac{{\lambda_i}^2}{\mu_i(\mu_i - \lambda_i)} + \frac{(\sum_{i=1}^{n} \lambda_i)^2}{\mu_{i+1}(\mu_{i+1} - \sum_{i=1}^{n} \lambda_i)}$$

3 Average time a patients spends in the system

$$W_s = \sum_{i=1}^{n} \frac{1}{\mu_i - \lambda_i} + \frac{1}{\mu_{i-1} - \sum_{i=1}^{n} \lambda_i}$$

4 Average waiting time of a patients in the queue

$$W_s = \frac{\lambda_i}{\mu_i(\mu_i - \lambda_i)} + \frac{\sum_{i=1}^{n} \lambda_i}{\mu_{i+1}(\mu_{i+1} - \sum_{i=1}^{n} \lambda_i)}$$

5 Traffic intensity

$$\rho_n = \frac{\lambda_n}{\mu_n}$$

The steady state general balance equation for the queuing system shows that $t \to \infty$ thus, the probability distribution $p_n(t)$ is no longer a function of t, and then t can be excluded. For $n = 0, p_{n-1}(t), \lambda_{n-1}$ and $\mu_n = 0$ (there must be a zero departure rate if there are no units present).

To determine the Probability distribution (p_n) for the number of patients in 44 NARHK, use p_n to compute in terms of ρ_0.

Therefore,

$$p_n = \prod_{i=1} \frac{\lambda_{i-1}}{\mu_i} \rho_0 \ \forall\ 1 \leq n \leq \infty$$

The proposed Structure shows that an extra couch is added to accommodate the incoming patients whose Kendall model is M/M/3:P/10/∞. Patients queue up at the treatment bay depending on the next available space. Thus, the service point 1, 2 and 3 have similar queues. This variable will assess the measurement of the system operating characteristics of 44 NARHK. The server for μ_2 and μ_3 in the existing model are maintained in the proposed model structure. Therefore, the Measurement of the system operating characteristics of NARHK which were analysed based on Ls (expected number in the system that is waiting and service, Lq (expected number waiting to be serve), Ws (expected time spent in the system), Wq (expected time spent waiting) and (ρ) is the traffic intensity are accessed with respect to the assumed values which will produce decreased results in the death rate.

4.0 RESULTS AND DISCUSSIONS

Here, results and discussion are presented. To illustrate the analysis, the modified Oladejo-Agashua model [] was applied as related to the existing and proposed model of 44

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NARHK.

4.1 Arrival time (λ)

$$\lambda_n = \frac{366}{24} = 15.25 \text{ Patients/day}$$

This suggests that an average of 15 (approx.) patients arrive every day.

4.2 Service time (μ)

$$\mu_n = \frac{42}{18} = 2.3333 \text{ patients/hour}$$

The inter-service time of the patients for the entire period

$$\frac{1}{2.3333} = 0.4285$$

Since our unit time is in days, we multiply through by 24 to obtain the average number of patients attended to per day, hence we have;

$$0.4285 \times 24 = 10.3 \text{ per day}$$

This suggests that an average of 10 (approx.) patients arrive every day.

Hence, our traffic intensity in the system is calculated as;

$$\rho_n = \frac{\lambda_n}{\mu_n} =$$

$$\frac{15.25}{10.3} \text{ per day}$$

$$\rho_n = 1.4805$$

Table 4.1; Analysis of daily arrival and service rate at the entire facility in A&E of 44 NARHK (Existing Facility)

QUEUE STATION	EXISTING EMERGENCY DEPARTMENT
Arrival Time per day λ	15.25
Service Time per day μ	10.3
Number of Servers S	2
Capacity of the System C	4
Efficiency of the system	0.6512
Queuing Type	M/M/2/4
Mean Number at System (Ls)	1.705772
Mean Time at System (Ws)	0.127165
Mean Number in Queue Awaiting Service (Lq)	0.403461
Mean Time in Queue Awaiting Service (Wq)	0.030078
Mean Number in Service being Served	1.30231
Mean Time in Service being Served	0.097087
Probability All Servers Idle	0.200452
Prob. All Servers Busy	0.502762
Prob. System Full	0.120407

Explanation of table 4.1

From table 4.1, it was observed that the efficiency of the model is very high. The implication of this is that the intensity at which the patients were been attended to is very low, the patients experience bottleneck and there is every possibility for patients turn over or diversion from the hospital. In the existing model there are at least on the average two patients on queue waiting for treatment in the system while there is at least one patient on queue waiting to be served. The probability that the entire server were idle also implies that for the period of active service none of the service was idle. The entire system was full and it experiences a bottle neck; hence the quality of service delivery to the patient is poor.

1.2 Proposed Structure

Table 4.2; Analysis of the proposed model in A&E of 44 NARHK (Proposed Facility)

QUEUE STATION	PROPOSED EMERGENCY DEPARTMENT
Arrival Time per day λ	15.25
Service Time per day μ	10.3
Number of Servers S	3
Capacity of the System C	4
Efficiency of the system	0.4635
Queuing Type	M/M/3/4
Mean Number at System (Ls)	1.45132
Mean Time at System (Ws)	0.10134
Mean Number in Queue Awaiting Service (Lq)	0.06089
Mean Time in Queue Awaiting Service (Wq)	0.00425
Mean Number in Service being Served	1.39043
Mean Time in Service being Served	0.09709
Probability All Servers Idle	0.22807
Prob. All Servers Busy	0.18426
Prob. System Full	0.06089

Explanation of table 4.2

From table 4.2, while extra server was added to the existing model it observed that the proposed model is more efficient than the existing structure and while we introduce just one extra server is due to fact that we are trying to consider the cost implication of the proposed model. The proposed model also save extra 0.025826 hours of the patients' time waiting for service in the system. With this proposed model, there is more room to accommodate more incoming patients, as the probability of system been full is much more lesser compared to the existing model and one extra server was added due to the operational cost implication.

5 CONCLUSIONS.

In this research, modified Oladejo-Agashua model was used to fashion out the new model for the ED in 44 NARHK by increasing the number of servers in the queuing system which has reduced the queue size and mean waiting time of patients in the facility. The efficiency of the proposed model has advantage over the existing model. Thus in the proposed model structure, all the model parameter values are advantageous over the existing structure. The Ws (Average time a patients spends in the system) in the proposed structure is greater that Ws (Average time a patients spends in the system) in the existing structure because there are less patients in the proposed structure that in the existing structure.

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